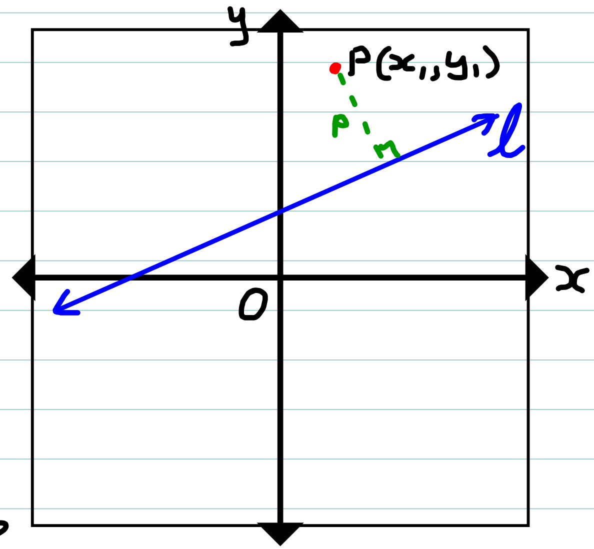
Distance From a Point to a Line (Perpendicular Distance)

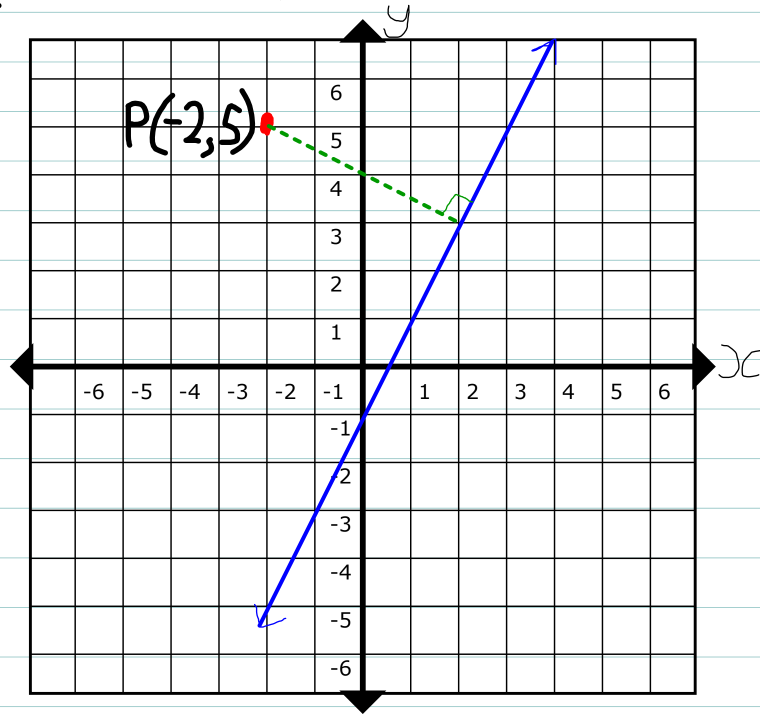
The perpendicular distance from the point (x_1, y_1) to the line $ax+by+c=0$ is

$$\boxed{p = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}}$$



Example Find the perpendicular distance from the point $P(-2,5)$ to the line $y=2x-1$.

(x_1,y_1)



First change the line to general form.

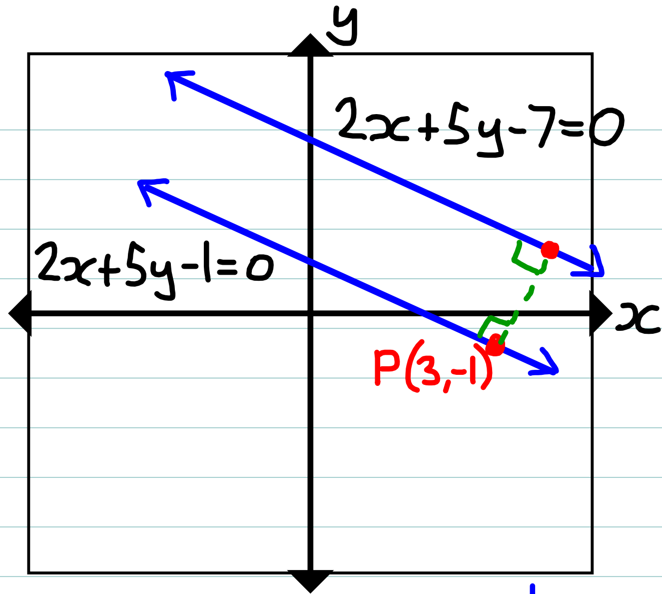
so, $\underset{a}{2x} \underset{b}{-y} \underset{c}{-1} = 0$

$$p = \frac{|2(-2)+(-1)(5)-1|}{\sqrt{2^2+(-1)^2}} = \frac{|-4-5-1|}{\sqrt{5}}$$

$$= \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5} \#$$

Distance Between Parallel Lines

Choose any point on one line and find its perpendicular distance to the other line.



Example

Find the perp. distance between the two parallel lines

$2x+5y-1=0$

and

$2x+5y-7=0$

Solution

$$\overset{a}{2}x+\overset{b}{5}y\overset{c}{-7}=0$$

$(\overset{x_1}{3},\overset{y_1}{-1})$

Choose any convenient point on the first line. [HOW?] We know now

CHOOSE (3,-1)

Now, the distance between the lines is the perpendicular from P to the second line.

$$p=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

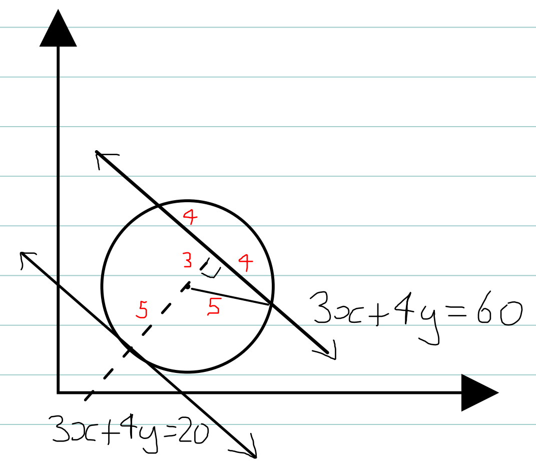
$$p=\frac{|2(3)+5(-1)-7|}{\sqrt{2^2+(5)^2}}$$

$$=\frac{|6-5-7|}{\sqrt{29}}=\frac{6}{\sqrt{29}}\times\frac{\sqrt{29}}{\sqrt{29}}$$

$$=\frac{6\sqrt{29}}{29}\ \#$$

Circles and the Perpendicular Formula

A line is tangent to a circle when its perpendicular distance from the centre is equal to the radius.



__Example__: Show that $\ell: 3x+4y-20=0$ is a tangent to the circle $(x-7)^2+(y-6)^2=25$

Well, the circle has centre $\boxed{(7,6)}$ and radius 5.

The distance from the line ℓ to the centre is

$$p=\frac{|3(7)+4(6)+(-20)|}{\sqrt{3^2+4^2}}$$

$$=\frac{|21+24-20|}{\sqrt{25}}=\frac{25}{5}=5,$$ which is the same as the radius of the circle, so the line is tangent to the circle.

__Example__: On the same circle, find the length of the chord cut off by the line $m: 3x+4y-60=0$.

Well, the distance p from the line m to the centre is

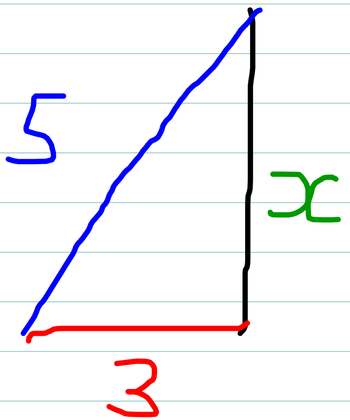
$$p=\frac{|21+24-60|}{\sqrt{3^2+4^2}}=3$$

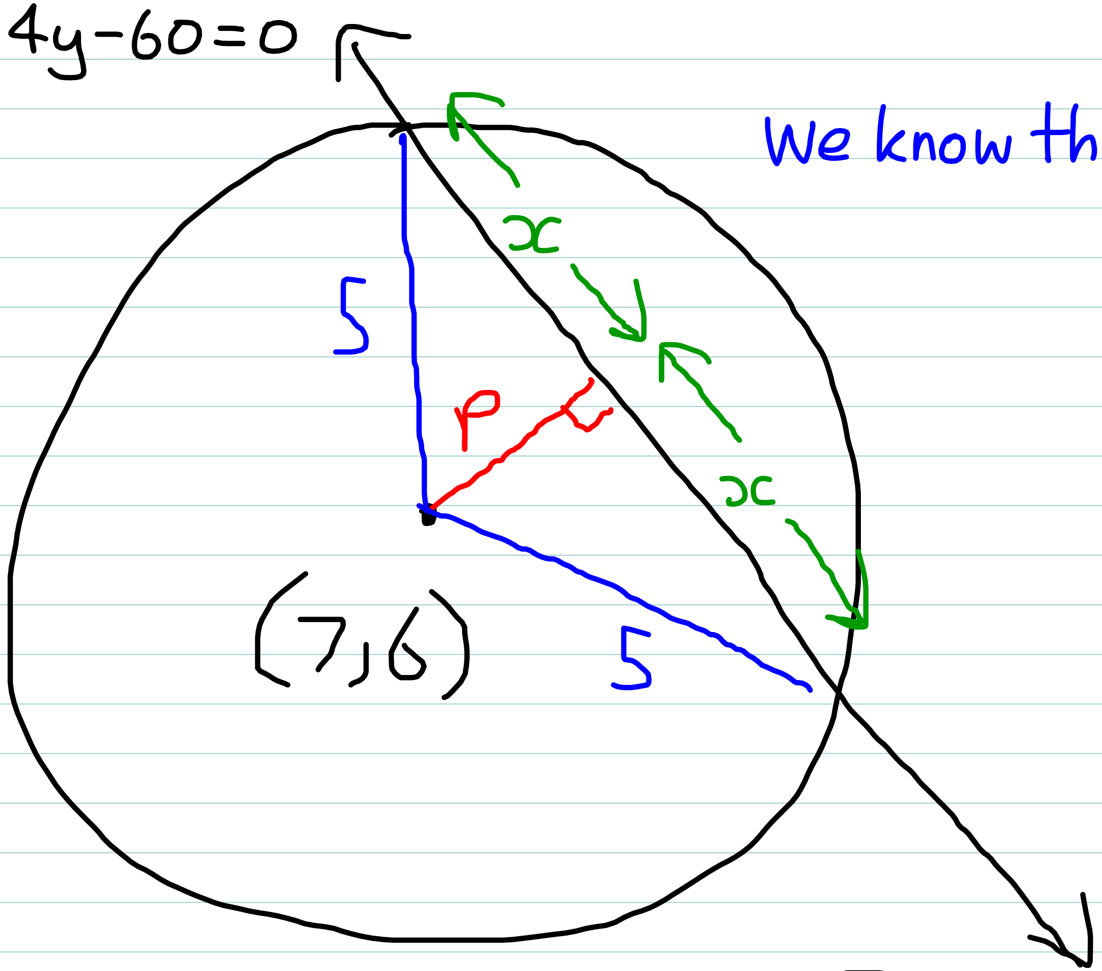
[SEE NEXT PAGE]

On the same circle find the length of the chord cut off by by the line $m: 3x+4y-60=0$

$$p = \frac{|3(7)+4(6)-60|}{\sqrt{3^2+4^2}} = 3$$

We know that r = radius $= 5$





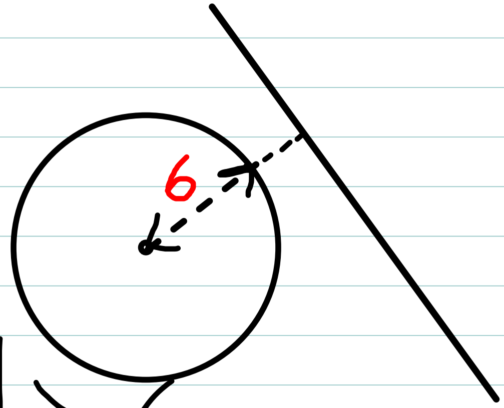
$m: 3x+4y-60=0$

By Pythagoras

$$x^2+3^2=5^2$$
$$x^2=16$$
$$x=4$$

The length of the chord is $2x = 8$ units #

Example: For what values of k will the line $5x-12y+k=0$ never intersect with the circle with centre $P(-3,1)$ and radius 6?



So, $\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} > 6$

$$\frac{|5(-3)+(-12)(1)+k|}{\sqrt{5^2+(-12)^2}} = \frac{|-15-12+k|}{\sqrt{25+144}}$$

$$= \frac{|-27+k|}{13} > 6$$

$$= |-27+k| > 78$$

$$= |k-27| > 78$$

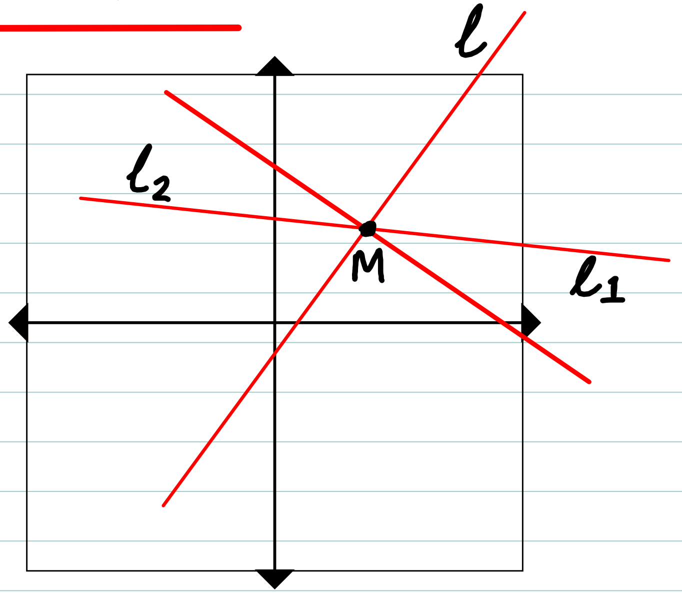
$\Rightarrow k<-51$ or $k>105$ #

$$\boxed{\begin{array}{lcl} & |k-27|>78 & \\ \Rightarrow\ k-27>78 & \text{AND} & k-27<-78 \\ k>78+27 & & k<-78+27 \\ k>105 & & k<-51 \end{array}}$$

Lines Through the Intersection of Two Given Lines

Suppose that two lines intersect at point M. Well, every line passing through point M has the form

$$(a_1x+b_1y+c_1)+k(a_2x+b_2y+c_2)=0$$



This is for next lesson.